

METHODOLOGY OF CONSTRAINED MULTIPLIER

Veronika Miřková, Vladimír Mlynarovič¹

The unconstrained multiplier, derived in previous research (Breinsinger, Thomas and Thurlow, 2009 and Mitkova, 2018) assumes fixed price set leading to changes in output as a reaction in changes in demand side of the economy. This assumption requires unlimited supply in each production sector, which is mostly unrealistic prerequisite. The input coefficients are fixed meaning the demand shocks have no influence to the structure and relationships among sectors. For simplicity the linkage effects are assumed to be linear. The model has final form $\mathbf{Z} = (\mathbf{I} - \mathbf{M})^{-1}\mathbf{E}$

Where:

- \mathbf{E} matrix of exogenous component of demand,
- \mathbf{M} coefficient matrix derived from the social accounting matrix by dividing each column through by its column totals,
- \mathbf{Z} matrix of total demand for each commodity.

This model was developed further by Breinsinger, Thomas and Thurlow (2009) to the constrained multiplier model for two sectors; we extended the model for n sectors in matrix form. The constrained multiplier model enriches the unconstrained one by dividing sectors to endogenous and exogenous. Sectors that can change the production level – the supply response is unconstrained, are treated as exogenous and the sectors with the supply constraints or fixed level of output as the endogenous sectors. All other previous assumptions regarding input coefficients and linkage effects hold.

Let:

- E_i exogenous component of demand for commodity $i, i = 1, 2, \dots, n$
- X_j gross output of activity $j, j = 1, 2, \dots, n$
- C_i household consumption of commodity $i, i = 1, 2, \dots, n$
- Y total household income (equal to total factor income)
- V_j factor income from activity $j, j = 1, 2, \dots, n$
- Z_{ij} intermediate demand for commodity i in activity $j, i, j = 1, 2, \dots, n$
- a_{ij} technical coefficients, $i, j = 1, 2, \dots, n$
- b_i share of domestic output in total demand, $i = 1, 2, \dots, n$
- c_i household consumption expenditure shares, $i = 1, 2, \dots, n$
- v_j share of value-added or factor income in gross output, $j = 1, 2, \dots, n$

Total demand of sector Z is composed of intermediate demand, final demand and exogenous demand:

$$\sum_{j=1}^n Z_{ij} + C_i + E_i = Z_i \quad i = 1, 2, \dots, n \quad (1)$$

¹ Miřková and Mlynarovič affiliated at Faculty of Social and Economic Sciences, Comenius University in Bratislava

where Z_i is total demand for commodity i .

If

$$a_{ij} = \frac{Z_{ij}}{X_j} \quad i = 1, 2, \dots, n$$

$$c_i = \frac{C_i}{\sum_{j=1}^n V_j} = \frac{C_i}{Y} \quad i = 1, 2, \dots, n$$

$$b_i = \frac{X_i}{Z_i} \quad i = 1, 2, \dots, n$$

$$v_j = \frac{V_j}{X_j} \quad j = 1, 2, \dots, n$$

$$\sum_{j=1}^n a_{ij} X_j + c_i Y + E_i = Z_i \quad i = 1, 2, \dots, n$$

Then (1) may be written as

$$\sum_{j=1}^n a_{ij} b_j Z_j + c_i \sum_{j=1}^n v_j b_j Z_j + E_i = Z_i, \quad i = 1, 2, \dots, n \quad (2)$$

$$Z_i - \sum_{j=1}^n a_{ij} b_j Z_j - c_i \sum_{j=1}^n v_j b_j Z_j = E_i, \quad i = 1, 2, \dots, n \quad (3)$$

Let's denote sectors $i, i = 1, 2, \dots, k$ as exogenous and sectors $i = k+1, k+2, \dots, n$ as endogenous, then system (3) may be divided to exogenous part in equations (4):

$$Z_i - \sum_{j=1}^k a_{ij} b_j Z_j - \sum_{j=1}^k c_i v_j b_j Z_j - \sum_{j=k+1}^n a_{ij} b_j Z_j - \sum_{j=k+1}^n c_i v_j b_j Z_j = E_i, \quad i = 1, \dots, k \quad (4a)$$

$$Z_i - \sum_{j=1}^k (a_{ij} b_j + c_i v_j b_j) Z_j - \sum_{j=k+1}^n (a_{ij} b_j + c_i b_j v_j) Z_j = E_i, \quad i = 1, \dots, k \quad (4b)$$

and for endogenous part in equations (5):

$$-\sum_{j=1}^k a_{ij}b_jZ_j - \sum_{j=1}^k c_iv_jb_jZ_j + Z_i - \sum_{j=k+1}^n a_{ij}b_jZ_j - \sum_{j=k+1}^n c_iv_jb_jZ_j = E_i, \quad i = k+1, \dots, n \quad (5a)$$

$$-\sum_{j=1}^k (a_{ij}b_j - c_iv_jb_j)Z_j + Z_i - \sum_{j=k+1}^n (a_{ij}b_j - c_iv_jb_j)Z_j = E_i, \quad i = k+1, \dots, n \quad (5b)$$

Let's divide matrices of demand \mathbf{E} and total demand \mathbf{Z} to endogenous (EN) and exogenous parts (EX) as follows:

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_{EX} \\ \mathbf{E}_{EN} \end{bmatrix}, \mathbf{E}_{EX} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_k \end{bmatrix}, \mathbf{E}_{EN} = \begin{bmatrix} E_{k+1} \\ E_{k+2} \\ \vdots \\ E_n \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{EX} \\ \mathbf{Z}_{EN} \end{bmatrix}, \mathbf{Z}_{EX} = \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_k \end{bmatrix}, \mathbf{Z}_{EN} = \begin{bmatrix} Z_{k+1} \\ Z_{k+2} \\ \vdots \\ Z_n \end{bmatrix}$$

Let

$$m_{ij} = a_{ij}b_j + c_iv_jb_j, \quad i, j = 1, \dots, n$$

and define matrices

$$\mathbf{M} = (m_{ij})_{n \times n}$$

$$[\mathbf{I}_k - \mathbf{M}_{kk}] = \begin{bmatrix} 1 - m_{11} & -m_{12} & \cdots & -m_{1k} \\ -m_{21} & 1 - m_{22} & \cdots & -m_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ -m_{k1} & -m_{k2} & \cdots & 1 - m_{kk} \end{bmatrix}$$

$$[\mathbf{I}_l - \mathbf{M}_{ll}] = \begin{bmatrix} 1 - m_{k+1,k+1} & -m_{k+1,k+2} & \cdots & -m_{k+1,n} \\ -m_{k+2,k+1} & 1 - m_{k+2,k+2} & \cdots & -m_{k+2,n} \\ \vdots & \vdots & \ddots & \vdots \\ -m_{n,k+1} & -m_{n,k+2} & \cdots & 1 - m_{n,n} \end{bmatrix}$$

$$[\mathbf{M}_{kl}] = \begin{bmatrix} -m_{1,k+1} & -m_{1,k+2} & \cdots & -m_{1,n} \\ -m_{2,k+1} & -m_{2,k+2} & \cdots & -m_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ -m_{k,k+1} & -m_{k,k+2} & \cdots & -m_{k,n} \end{bmatrix}$$

$$[\mathbf{M}_{lk}] = \begin{bmatrix} -m_{k+1,1} & -m_{k+1,2} & \cdots & -m_{k+1,k} \\ -m_{k+2,1} & -m_{k+2,2} & \cdots & -m_{k+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ -m_{n,1} & -m_{n,2} & \cdots & -m_{n,k} \end{bmatrix}$$

where $l = n - k$. Then equation (3) can be written in the matrix form

$$(\mathbf{I} - \mathbf{M})\mathbf{Z} = \mathbf{E} \quad (4)$$

Equations (5) and (6) re-written in matrix form

$$\begin{bmatrix} \mathbf{I}_k - \mathbf{M}_{kk} & \mathbf{M}_{kl} \\ \mathbf{M}_{lk} & \mathbf{I}_l - \mathbf{M}_{ll} \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{EX} \\ \mathbf{Z}_{EN} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{EX} \\ \mathbf{E}_{EN} \end{bmatrix} \quad (7)$$

Then the multiplier is

$$\begin{bmatrix} \mathbf{Z}_{EX} \\ \mathbf{Z}_{EN} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_k - \mathbf{M}_{kk} & \mathbf{M}_{kl} \\ \mathbf{M}_{lk} & \mathbf{I}_l - \mathbf{M}_{ll} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{E}_{EX} \\ \mathbf{E}_{EN} \end{bmatrix} \quad (8)$$

A change in an exogenous demand in unconstrained sector leads to a final change in total demand in this sector, including forward and backward linkages. In the constrained sector the demand changes lead to proportion changes in domestic and rest of the world production.

Acknowledgment

This paper research was supported by the Slovak Research and Development Agency, project No. PP-20-0026: Zotavíme sa z pandémie COVID-19: sociálne, ekonomické a právne perspektívy pandemickej krízy

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